Scale Invariance, Inflation and the Present Vacuum Energy of the Universe

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The possibility of mass in the context of scale-invariant, generally covariant theories, is discussed. Scale invariance is considered in the context of a gravitational theory where the action, in the first order formalism, is of the form $S = \int L_1 \Phi d^4x + \int L_2 \sqrt{-g} d^4x$ where Φ is a density built out of degrees of freedom independent of the metric. For global scale invariance, a "dilaton" ϕ has to be introduced, with non-trivial potentials $V(\phi) = f_1 e^{\alpha\phi}$ in L_1 and $U(\phi)$ $= f_2 e^{2\alpha\phi}$ in L_2. This leads to non-trivial mass generation and a potential for ϕ which is interesting for inflation. The model after ssb can be connected to the induced gravity model of Zee, which is a successful model of inflation. Models of the present universe and a natural transition from inflation to a slowly accelerated universe at late times are discussed.

1 The Model in the absence of fermions

The concept of scale invariance appears as an attractive possibility for a fundamental symmetry of nature. In its most naive realizations, such a symmetry is not a viable symmetry, however, since nature seems to have chosen some typical scales.

Here we will find that scale invariance can nevertheless be incorporated into realistic, generally covariant field theories. However, scale invariance has to be discussed in a more general framework than that of standard generally relativistic theories, where we must allow in the action, in addition to the ordinary measure of integration $\sqrt{-g}d^4x$, another one, Φd^4x, where Φ is a density built out of degrees of freedom independent of the metric.

For example, given 4-scalars φ_a (a = 1,2,3,4), one can construct the density

$$\Phi = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d \tag{1}$$

One can allow both geometrical objects to enter the theory and consider [1]

$$S = \int L_1 \Phi d^4x + \int L_2 \sqrt{-g} d^4x \tag{2}$$

Here L_1 and L_2 are φ_a independent. There is a good reason not to consider mixing of Φ and $\sqrt{-g}$, like for example using $\frac{\Phi^2}{\sqrt{-g}}$. This is because (2) is invariant (up to the inte divergence) under the infinite dimensional symmetry $\varphi_a \to \varphi_a + f_a(L_1)$ where $f_a(L_1)$ is an arbitrary function of L_1 if L_1 and L_2 are φ_a independent. Such symmetry (up to the integral of a total divergence) is absent if mixed terms are present.

We will study now the dynamics of a scalar field ϕ interacting with gravity as given by the action (2) with [2,3,4]

$$L_1 = \frac{-1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi), L_2 = U(\phi) \tag{3}$$

$$R(\Gamma, g) = g^{\mu\nu} R_{\mu\nu}(\Gamma), R_{\mu\nu}(\Gamma) = R^\lambda_{\mu\nu\lambda}, R^\lambda_{\mu\nu\sigma}(\Gamma) = \Gamma^\lambda_{\mu\nu,\sigma} - \Gamma^\lambda_{\mu\sigma,\nu} + \Gamma^\lambda_{\alpha\sigma}\Gamma^\alpha_{\mu\nu} - \Gamma^\lambda_{\alpha\nu}\Gamma^\alpha_{\mu\sigma}. \tag{4}$$

In the variational principle $\Gamma^\lambda_{\mu\nu}, g_{\mu\nu}$, the measure fields scalars φ_a and the scalar field ϕ are all to be treated as independent variables. If we perform the global scale transformation ($\theta =$ constant)

$$g_{\mu\nu} \to e^\theta g_{\mu\nu} \tag{5}$$

then (2), with the definitions (3), (4), is invariant provided $V(\phi)$ and $U(\phi)$ are of the form

$$V(\phi) = f_1 e^{\alpha\phi}, U(\phi) = f_2 e^{2\alpha\phi} \tag{6}$$

and φ_a is transformed according to $\varphi_a \to \lambda_a \varphi_a$ (no sum on a) which means $\Phi \to \left(\prod_a \lambda_a\right) \Phi \equiv \lambda \Phi$ such that $\lambda = e^\theta$ and $\phi \to \phi - \frac{\theta}{\alpha}$. In this case we call the scalar field ϕ needed to implement scale invariance "dilaton".

1.1 Equations of Motion

Let us consider the equations which are obtained from the variation of the φ_a fields. We obtain then $A^\mu_a \partial_\mu L_1 = 0$ where $A^\mu_a = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d$. Since det $(A^\mu_a) = \frac{4^{-4}}{4!} \Phi^3 \neq 0$ if $\Phi \neq 0$. Therefore if $\Phi \neq 0$ we obtain that $\partial_\mu L_1 = 0$, or that $L_1 = M$, where M is constant. This constant M appears in a self-consistency condition of the equations of motion that allows us to solve for $\chi \equiv \frac{\Phi}{\sqrt{-g}}$

$$\chi = \frac{2U(\phi)}{M + V(\phi)}. \tag{7}$$

To get the physical content of the theory, it is convenient to go to the Einstein conformal frame where

$$\overline{g}_{\mu\nu} = \chi g_{\mu\nu} \tag{8}$$

and χ given by (7). In terms of $\overline{g}_{\mu\nu}$ the non Riemannian contribution (defined as $\Sigma^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \{^\lambda_{\mu\nu}\}$ where $\{^\lambda_{\mu\nu}\}$ is the Christoffel symbol), disappears from the equations, which can be written then in the Einstein form ($R_{\mu\nu}(\overline{g}_{\alpha\beta}) =$ usual Ricci tensor)

$$R_{\mu\nu}(\overline{g}_{\alpha\beta}) - \frac{1}{2} \overline{g}_{\mu\nu} R(\overline{g}_{\alpha\beta}) = \frac{\kappa}{2} T^{eff}_{\mu\nu}(\phi) \tag{9}$$

where

$$T^{eff}_{\mu\nu}(\phi) = \phi_{,\mu}\phi_{,\nu} - \frac{1}{2}\overline{g}_{\mu\nu}\phi_{,\alpha}\phi_{,\beta}\overline{g}^{\alpha\beta} + \overline{g}_{\mu\nu} V_{eff}(\phi), V_{eff}(\phi) = \frac{1}{4U(\phi)}(V+M)^2. \tag{10}$$

If $V(\phi) = f_1 e^{\alpha\phi}$ and $U(\phi) = f_2 e^{2\alpha\phi}$ as required by scale invariance, we obtain from (10)

$$V_{eff} = \frac{1}{4f_2}(f_1 + Me^{-\alpha\phi})^2 \tag{11}$$

Since we can always perform the transformation $\phi \to -\phi$ we can choose by convention $\alpha > O$. We then see that as $\phi \to \infty, V_{eff} \to \frac{f_1^2}{4f_2} =$ const. providing an infinite flat region. Also a minimum is achieved at zero cosmological constant , without fine tuning for the case $\frac{f_1}{M} < O$ at the point $\phi_{min} = \frac{-1}{\alpha} ln \mid \frac{f_1}{M} \mid$. Finally, the second derivative of the potential V_{eff} at the minimum is $V''_{eff} = \frac{\alpha^2}{2f_2} \mid f_1 \mid^2 > O$

2 Interpretations, Generalizations and Physical Applications of the Model

There are many interesting issues that one can raise here. The first one is of course the fact that a realistic scalar field potential, with massive excitations when considering the true vacuum state, is achieved in a way consistent with the idea of scale invariance. An interesting point to be made concerning this is that even though spontaneous symmetry breaking has taken place, no Goldstone boson appears nevertheless [4] when analyzing the theory in its ground state. This interesting and unusual effect is due to the fact that although a locally conserved current can be defined (from Noether's theorem), this still does not lead to a globally conserved charge, because the currents have an infrared singular behavior that causes scale charge to leak to infinity [4].

The second point to be raised is that since there is an infinite region of flat potential for $\phi \to \infty$, we expect a slow rolling inflationary scenario to be viable, provided the universe is started at a sufficiently large value of the scalar field ϕ for example. It is also very interesting to notice that the theory can be related to the induced gravity theory of Zee [5], defined by the action,

$$S = \int \sqrt{-g}(-\frac{1}{2}\epsilon\varphi^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{\lambda}{8}(\varphi^2 - \eta^2)^2)d^4x \tag{12}$$

Here it is assumed that the second order formalism is used, i.e. $R = R(g) =$ usual Riemannian scalar curvature defined in terms of $g_{\mu\nu}$. Notice that if $\eta = 0$, the action is invariant under the global scale transformation $g_{\mu\nu} \to e^{\theta} g_{\mu\nu}$, $\varphi \to e^{-\frac{\theta}{2}}\varphi$, but a finite induced Newton's constant is defined only if η is non vanishing. Then defining $(2k^2 = \kappa)$ $\overline{g}_{\mu\nu} = k^2\epsilon\varphi^2 g_{\mu\nu}$ and the scalar field $\phi = \frac{1}{k}\sqrt{6 + \frac{1}{\epsilon}} ln\varphi$, one can then show that the induced gravity model is equivalent to standard General Relativity (expressed in terms of $\overline{g}_{\mu\nu}$) minimally coupled to the scalar field ϕ which has a potential [6] , $V_{eff} = \frac{\lambda}{8k^4\epsilon^2}(1 - \eta^2 e^{-2\sqrt{\frac{\epsilon}{1+6\epsilon}}k\phi})^2$ which is exactly the form (11) with $\alpha = 2\sqrt{\frac{\epsilon}{1+6\epsilon}}k$ (in Ref.6, k is called κ). The induced gravity model (12) is quite successful from the point of view of its applications to inflation and it has been studied by a number of authors in this context [7]. Notice that the induced gravity model is not consistent with scale invariance for a non vanishing η, while the theory developed here, which leads to the induced gravity model after ssb, has been constructed starting with scale invariance as a fundamental principle.

Furthermore, one can consider this model as suitable for the present day universe rather than for the early universe, after we suitably reinterpret the meaning of the scalar field ϕ. This can provide a long lived almost constant vacuum energy for a long period of time, which can be small if $f_1^2/4f_2$ is small. Such small energy density will eventually disappear when the universe achieves its true vacuum state.

Notice that a small value of $\frac{f_1^2}{f_2}$ can be achieved if we let $f_2 >> f_1$. In this case $\frac{f_1^2}{f_2} << f_1$, i.e. a very small scale for the energy density of the universe is obtained by the existence of a very high scale (that of f_2) the same way as a small fermion mass is obtained in the see-saw

mechanism [8] from the existence also of a large mass scale. It can be shown also that if we take $f_2 >> f_1$, so that the vacuum energy is small, this also produces a drastic suppression in the fermion particle masses (everything can be done consistently with scale invariance, see Ref.3) and a possible correlation between the lightest particle mass and the vacuum energy of the universe is argued for (see the transparencies of this talk which can be found in the web site of this conference). The scale invariant way of introducing fermion masses has also implications concerning the "cosmic coincidences" problem (see Ref.3).

Finally, this kind of theories can naturally provide a dynamics that interpolates between a high energy density (associated with inflation) and a very low energy density (associated with the present universe). For this consider two scalar fields ϕ_1 and ϕ_2, with normal kinetic terms coupled to the measure Φ as it has been done with the simpler model of just one scalar field. Introducing for ϕ_1 a potential $V_1(\phi_1) = a_1 e^{\alpha_1 \phi_1}$ that couples to Φ and another $U_1(\phi_1) = b_1 e^{2\alpha_1 \phi_1}$ that couples to $\sqrt{-g}$ as required by scale invariance and the potential for ϕ_2, $V_2(\phi_2) = a_2 e^{\alpha_2 \phi_2}$ that couples to Φ and another $U_2(\phi_2) = b_2 e^{2\alpha_2 \phi_2}$ that couples to $\sqrt{-g}$, we arrive (after going through the same steps as those explained in the model with just one scalar, i.e. solving the constraint and going to the Einstein frame) at the the effective potential

$$V_{eff} = \frac{(V_1(\phi_1) + V_2(\phi_2) + M)^2}{4(U_1(\phi_1) + U_2(\phi_2))} \tag{13}$$

which introduces interactions between ϕ_1 and ϕ_2, although no interactions appeared in the original action (i.e. no direct couplings appeared). If we take then $\alpha_1 \phi_1$ very big while ϕ_2 is fixed, then V_{eff} approaches the constant value $\frac{a_1^2}{4b_1}$ while if we take $\alpha_2 \phi_2$ to be very big while ϕ_1 is kept fixed, then V_{eff} approaches the constant value $\frac{a_2^2}{4b_2}$. One of these flat regions of the potential can be associated with a very high energy density, associated with inflation and the other can be very small and associated with the energy density of the present universe. The effective potential (13) provides therefore a dynamics that interpolates naturally between the inflationary phase and the present slowly accelerated universe.

References

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